Morning Madness - Solution

The answer is $M = \frac{(2n)!}{2^n}$.
Denote by F_i the sock that Mike puts on on foot i for $1 \leq i \leq n$. Denote S_i analoguously for the shoes. Now we can denote a correct oredering by a sequence of F_i and S_i. For example, if $n = 2$, a correct ordering is $F_1F_2S_2S_1$. Let V_0 be the set of any orderings of these $2n$ symbols. For any such ordering $\sigma \in V_0$, denote by $\sigma(F_i)$ the index whose symbol is F_i. It should be clear that $|V_0| = (2n)!$. Let $V_1 = \{\sigma \in V_0 : \sigma(F_1) < \sigma(S_1)\}$ be the set of orderings such that Mike has put the sock on foot 1 before he puts a shoe on foot 1. Now the function $\phi : V_1 \to V_0 \setminus V_1$ given by

$$\phi(\sigma)(i) = \begin{cases} S_1 & \text{if } \sigma(i) = F_1; \\ F_1 & \text{if } \sigma(i) = S_1; \\ \sigma(i) & \text{else,} \end{cases}$$

i.e. by switching S_1 and F_1 in the ordering, is clearly a bijection. Hence, $|V_1| = |V_0|/2$.
Now letting $V_j = \{\sigma \in V_{j-1} : \sigma(F_j) < \sigma(S_j)\}$ be the set such that the first j socks are put on in the right order, we see that analoguously to the argument above we have $|V_j| = |V_{j-1}|/2$. Hence

$$M = |V_n| = \frac{|V_0|}{2^n} = \frac{(2n)!}{2^n},$$

as wanted.